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Hawking Radiation as Tunneling from Garfinkle-Horowitz-Stromingen Dilaton Black Hole

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Abstract Applying quantum tunneling method, this paper has studied the Hawking radiation of Garfinkle-Horowitz-Stromingen dilaton black hole. In this way, the emission rates of massless particles and massive particles tunneling across the event horizon of black holes is obtained. The result shows that the radiation spectrum of these two different kinds of outgoing particles is related to the change of Bekenstein-Hawking entropy, which is no longer precisely thermal.

Keywords Garfinkle-Horowitz-Stromingen dilaton black hole · Tunneling rate · Energy conservation · Bekenstein-Hawking entropy

1 Introduction

In 1974, Hawking discovered that black holes have thermal radiation and the radiation spectrum is precisely thermal after taking the fixed space-time background of the black hole into account [1]. Hawking radiation will give rise to the information loss paradox of black holes physics. If one thought that the information is lost in black hole evaporation, a pure quantum state will be converted into a mixed one, which disobeys an underlying unitary theory. Recently, Parikh and Wilczek presented a semi-classical method, which describes the Hawking radiation as a tunneling process, when the energy conservation and the self-gravitation interaction are considered [2]. Applying this method, the radiation spectrum of massless particles tunneling across both the static Schwarzschild black hole and Reissner-Nordström black hole is obtained, and this Hawking radiation spectrum is not precisely thermal. Therefore, according to Parikh-Wilczek's method, the information is not lost and can get out of the black hole. Motivated by Parikh-Wilczek's work, researchers have studied

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Hawking radiation of other black holes [3–14]. All the obtained results support Parikh-Wilczek's viewpoint. Liu [15] has obtained the same result when the reaction of the radiation to the space-time background and the Damour-Ruffini method are considered [16], that is, the factual radiation is not precisely thermal but subtle correction to the Hawking precisely thermal spectrum. Considering the energy conservation, charge conservation and self-gravitation interaction, Jiang et al. [17] generalized the complex path integral method to study the radiation spectrum of the Reissner-Nordström-anti-de Sitter black hole and obtained the same result to Parikh's. In order to make the result more meaningful, radiation spectrum of Garfinkle-Horowitz-Stromingen dilaton black hole with special topological structure will be studied. Such black holes hold the mass, charge and the axial dilaton charge determined by the coupling of the mass and charge. For a general black hole, we consider that the outgoing particles can be massless or charged particles too. In this paper, we will study the emission rate of massless particles and massive particles tunneling across the event horizon of Garfinkle-Horowitz-Stromingen dilaton black hole, and the obtained radiation spectrum is not precisely thermal but subtle correction to the Hawking precisely thermal spectrum.

2 The Space-Time of Garfinkle-Horowitz-Stromingen Dilaton After the Painlevè Coordinate Transformation

In natural unit, adopted the $(+,-,-,-)$ signature, the space-time element of Garfinkle-Horowitz-Stromingen dilaton black hole can be expressed as [18–20]

$$ds^2 = \left(1 - \frac{2M}{r-D}\right)dt_G^2 - \left(1 - \frac{2M}{r-D}\right)^{-1} dr^2 - (r^2 - D^2)(d\theta^2 + \sin^2\theta d\phi^2), \qquad (1)$$

where $D = -\frac{Q^2}{2M}$, D is axial dilaton charge, M the mass of black hole, Q the charge of black hole, and t_G is the time coordinate of the space-time of Garfinkle-Horowitz-Stromingen dilaton black hole. In addition, this space-time is spherically symmetric. The equation $g_{00} = (1 - \frac{2M}{r-D}) = 0$ is just the event horizon equation of the black hole, so the event horizon radius is

$$r_h = 2M + D. \qquad (2)$$

Since the area of event horizon is $A_h = 4\pi(r_h^2 - D^2)$, setting $R^2 = r^2 - D^2$, the (1) can be written as

$$ds^2 = \left[1 - \frac{2M(\sqrt{R^2+D^2}+D)}{R^2}\right]dt_G^2 - \left[1 - \frac{2M(\sqrt{R^2+D^2}+D)}{R^2}\right]^{-1} \frac{R^2}{R^2+D^2}dR^2$$
$$- R^2(d\theta^2 + \sin^2\theta d\phi^2). \qquad (3)$$

It can be seen from (3) that there is coordinator singularity on the event horizon. In order to study the quantum tunneling on the event horizon, the coordinate singularity must be eliminated. So we make the Painlevè coordinate transformation [21]

$$t_G = t + F(R). \qquad (4)$$

The differential form of (4) is

$$dt_G = dt + F'(R)dR, \qquad (5)$$

where $F'(R)=\frac{dF(R)}{dR}$, and setting

$$G(R)=\frac{2M(\sqrt{R^2+D^2}+D)}{R^2}, \tag{6}$$

then (3) can be written as

$$\begin{aligned} ds^2 &= [1-G(R)]dt^2+2[1-G(R)]F'(R)dtdR \\ &\quad -\left\{\frac{R^2}{(R^2+D^2)[1-G(R)]}-[1-G(R)]F'^2(R)\right\}dR^2-R^2(d\theta^2+\sin^2\theta d\phi^2). \end{aligned} \tag{7}$$

Considering that the metric is flat Euclidean in radial to the constant-time slices, one can set

$$\frac{R^2}{(R^2+D^2)[1-G(R)]}-[1-G(R)]F'^2(R)=1, \tag{8}$$

and then the space-time elements of Garfinkle-Horowitz-Strominger dilaton black hole after the Painlevè coordinate transformation can be expressed as

$$ds^2=[1-G(R)]dt^2-2\sqrt{\frac{G(R)(R^2+D^2)-D^2}{R^2+D^2}}dtdR-dR^2-R^2(d\theta^2+\sin^2\theta d\phi^2). \tag{9}$$

Therefore, the coordinate singularity on the event horizon can be eliminated adopted the Painlevè coordinate system. In this coordinate, the event horizon coincides with the infinite red shift surface and the space-time is flat Euclidean in radial direction. These characteristics can offer much convenient to study quantum tunneling of black holes.

3 Tunneling Radiation of Uncharged Particles

3.1 Tunneling Radiation of Massless Particles

According to (9), the null radial geodesic of the massless particles can be represented as

$$\dot{R}=\frac{dR}{dt}=\pm\sqrt{\frac{R^2}{R^2+D^2}}-\sqrt{G(R)-\frac{D^2}{R^2+D^2}}, \tag{10}$$

where the $+$ $(-)$ sign can be identified with outgoing (ingoing) radial motion.

Considering the energy conservation, when the total mass of space-time is fixed and the black hole mass is allowed to varied, the mass parameter of the black hole M in (9) and (10) should be replaced by $M-\omega$ when the particle with positive energy ω tunnels out. In [9], they considered the massless particles as a shell of energy ω. As described above, the event horizon and the infinite red-shift surface are located in the same place. So the S wave of this shell has infinite blue shift near the horizon. Taking the infinite blue-shift at the event horizon into account, and considering that the wave-length is arbitrarily short, so geometric optics limit is reliable here. And then we can use the particle language to describe the outgoing of these S wave. In the semi-classical limit, applying the Wentzel-Kramers-Brillouin approximation, the relation between the emission rate Γ and the imaginary part of the particle action can be written as [22]

$$\Gamma\sim e^{-2\,\mathrm{Im}\,I}, \tag{11}$$

whose imaginary part of the action can be represented as

$$\mathrm{Im}\, I = \mathrm{Im} \int_{R_i}^{R_f} P_R dR = \mathrm{Im} \int_{R_i}^{R_f} \int_0^{P_R} dP' dR, \tag{12}$$

where P_R is the canonical momentum conjugate to R, R_i and R_f respectively represent the location of the event horizon before and after the particles emission, and they are often regarded as the two turning points of the tunneling potential hill. According to the Hamilton canonical equation, we get

$$\dot{R} = \frac{dH}{dP_R} = \frac{d(M-\omega)}{dP_R} = -\frac{d\omega}{dP_R}. \tag{13}$$

Substituting (13) into (12), and setting $D = -\frac{Q^2}{2(M-\omega)}$, we integrate over ω first and obtain

$$\mathrm{Im}\, I = \mathrm{Im} \int_0^{\omega} \int_{R_i}^{R_f} \frac{dR}{\sqrt{\frac{R^2}{R^2+D^2}} - \sqrt{G(R) - \frac{D^2}{R^2+D^2}}} d(-\omega') = -\frac{\pi}{2}(R_f^2 - R_i^2). \tag{14}$$

Substituting (14) into (11), the tunneling rate of massless particles is therefore

$$\Gamma \sim e^{-2\,\mathrm{Im}\, I} = e^{\pi(R_f^2 - R_i^2)} = e^{\frac{1}{4}(A_f - A_i)} = e^{\Delta S_{B-H}}, \tag{15}$$

where $R_i = 2M\sqrt{1 - \frac{Q^2}{2M^2}}$, $R_f = 2(M-\omega)\sqrt{1 - \frac{Q^2}{2(M-\omega)^2}}$, A_i and A_f respectively represent the area of event horizon before and after a particle emission, ΔS_{B-H} is the change of Bekenstein-Hawking entropy. Obviously, the radiation spectrum of massless particles is not precisely thermal.

3.2 Tunneling Radiation of Massive Particles

In quantum mechanics, tunneling process is instantaneous. The geodesics of massive particles are not light-like. For the convenience, we can consider the outgoing massive particles as spherical de Broglie wave [9]. Since the space-time of Garfinkle-Horowitz-Stromingen dilaton is stationary and the corresponding space-time slice is European, the Schrödinger equation holds also. Based on the Wentzel-Kramers-Brillouin approximation, the approximation wave equation is [7, 11]

$$\psi(R,t) = Ce^{i(\int_{R_i-\varepsilon}^{R} P_R dR - \omega t)}, \tag{16}$$

where $R_i - \varepsilon$ is the initial location of particles, P_R is the canonical momentum conjugate to R, ω is the positive energy of emission particles. Given a special ϕ_0, setting

$$\int_{R_i-\varepsilon}^{R} P_R dR - \omega t = \phi_0, \tag{17}$$

we can have

$$\frac{dR}{dt} = \dot{R} = \frac{\omega}{k}, \tag{18}$$

where k is the wave number of de Broglie wave, $\dot{R}$ is the phase velocity, the corresponding group velocity is

$$\upsilon_g = \frac{dR_c}{dt} = \frac{d\omega}{dk}, \tag{19}$$

where R_c expresses the location of particles, dR_c the corresponding displacement. According to de Broglie hypothesis, the relation between the group velocity υ_g and phase velocity υ_p of de Broglie wave is

$$\dot{R} = \upsilon_p = \frac{1}{2}\upsilon_g. \tag{20}$$

Based on Parikh's quantum tunneling, the two events of particles ingoing and outgoing the potential hill take place simultaneously. So according to Landau's theory of the coordinate clock synchronization, in a space-time decomposed in $(3+1)$, the coordinate time difference of this two events that take place simultaneously in difference places is [23]

$$\Delta T = -\int \frac{g_{0i}}{g_{00}} dx^i, \quad (i = 1, 2, 3). \tag{21}$$

If the simultaneity of coordinate clocks can be transmitted from one place to another and has nothing to do with the integration path, components of the metric should satisfy [24]

$$\frac{\partial}{\partial x^j}\left(-\frac{g_{0i}}{g_{00}}\right) = \frac{\partial}{\partial x^i}\left(-\frac{g_{0j}}{g_{00}}\right), \quad (i, j = 1, 2, 3). \tag{22}$$

Obviously, the line element (9) satisfies condition (22), that is, the coordinate clocks can be transmitted from one place to another though line element is not diagonal. This characteristic is important to the following study. From (21), we can obtain the difference of the coordinate times of these two simultaneous events

$$dt = -\frac{g_{0i}}{g_{00}} dx^i = -\frac{g_{01}}{g_{00}} dR_c, \quad (d\theta = d\phi = 0). \tag{23}$$

Then the group velocity is

$$\upsilon_g = \frac{dR_c}{dt} = -\frac{g_{00}}{g_{01}}. \tag{24}$$

From (9), (20) and (24), one can obtain

$$\upsilon_p = \dot{R} = -\frac{g_{00}}{2g_{01}} = \frac{1 - G(R)}{2\sqrt{G(R) - \frac{D^2}{R^2+D^2}}}. \tag{25}$$

Considering the self-gravitation, and when a positive energy ω particle tunnels out of black holes, (25) should be expressed as

$$\dot{R} = \frac{R^2 + Q^2 - 2(M-\omega)\sqrt{R^2 + \frac{Q^4}{4(M-\omega)^2}}}{2R^2\sqrt{\frac{2(M-\omega)\sqrt{R^2+\frac{Q^4}{4(M-\omega)^2}} - Q^2}{R^2} - \frac{Q^4}{4(M-\omega)^2 R^2 + Q^4}}}. \tag{26}$$

The tunneling rate Γ of massive particles and the imaginary part of the particle action satisfy the (11) and (12). In order to calculate (12), applying the Hamilton equation and the integral for dP_R changing to dH, then from (11), (12) and (26), we can obtain

$$\begin{aligned}\operatorname{Im} I &= \operatorname{Im}\int_0^{\omega}\int_{R_i}^{R_f}\frac{2R^2\sqrt{\frac{2(M-\omega)\sqrt{R^2+\frac{Q^4}{4(M-\omega)^2}}-Q^2}{R^2}-\frac{Q^4}{4(M-\omega)^2R^2+Q^4}}\,dR}{R^2+Q^2-2(M-\omega)\sqrt{R^2+\frac{Q^4}{4(M-\omega)^2}}}d(-\omega') \\ &= -\frac{\pi}{2}(R_f^2-R_i^2). \end{aligned} \tag{27}$$

Substituting (27) into (11), the tunneling rate of massive particles on event horizon is

$$\Gamma \sim e^{-2\operatorname{Im} I} = e^{\pi(R_f^2-R_i^2)} = e^{\frac{1}{4}(A_f-A_i)} = e^{\Delta S_{B-H}}. \tag{28}$$

Therefore, the tunneling rate of massive particles has the same form function to massless particles. It is evident that the practical radiation spectrum is not precisely thermal.

4 Tunneling of Charged Particles

Because Garfinkle-Horowitz-Strominger dilaton black holes hold charge, we can consider the situation that the emission particles may be charge. Under this condition, the geodesic of charged particles is not light-like, but time-like. The total mass of space-time is fixed and the black hole mass is allowed to varied. Taking the energy conservation, charge conservation and self-gravitation into account, when the particle with charge q and energy ω, the mass and charge of black hole will be replace by $M-\omega$ and $Q-q$, respectively. In addition, when we study the tunneling characteristics for charged particles, the electromagnetic field effect should be considered. Because the space-time of the black hole is spherically symmetric and has axial dilaton charges, the four-dimensional electromagnetic potential is

$$A_\mu = (A_t, 0, 0, 0), \tag{29}$$

where $A_t = -\frac{Q}{r-D} = -\frac{Q}{\sqrt{R^2+D^2}-D}$. The imaginary part of charged particles action can be written as

$$\operatorname{Im} I = \operatorname{Im}\int_{t_i}^{t_f}(P_R\dot{R} - P_A\dot{A})dt = \operatorname{Im}\int_{t_i}^{t_f}\left[\int_{(0,0)}^{(P_R,P_{A_R})}(\dot{R}dP'_R - \dot{A}_t dP'_{A_R})\right]\frac{dR}{\dot{R}}, \tag{30}$$

where P_R, P_{A_R} are the generalized momentums conjugate to the coordinator R_R and A_R, respectively. Applying the Hamilton equation, one can obtain

$$\begin{aligned}\dot{R} &= \frac{dH}{dP_R}\bigg|_{(R;A_t,P_{A_t})} = \frac{d(M-\omega)}{dP_R}, \\ \dot{A}_t &= \frac{dH}{dP_{A_R}}\bigg|_{(A_R;R,P_R)} = \frac{Q-q}{\sqrt{R^2+D^2}-D}\frac{d(Q-q)}{dP_{A_r}}. \end{aligned} \tag{31}$$

Substituting (31) and (26) into (30), the imaginary part of charged particles action can be obtained

$$\begin{aligned}\operatorname{Im} I &= \operatorname{Im}\int_{R_i}^{R_f}\int_{(M,Q)}^{(M-\omega,Q-q)}\left[d(M-\omega') - \frac{Q-q}{\sqrt{R^2+D^2}-D}d(Q-q')\right]\frac{dR}{\dot{R}} \\ &= -\pi\int_{(M,Q)}^{(M-\omega,Q-q)}[4Md(M-\omega') - 2Qd(Q-q')] = -\frac{\pi}{2}(R_f^2 - R_i^2). \end{aligned} \quad (32)$$

Then substituting (32) into (11), the tunneling rate of charged particles on event horizon is

$$\Gamma \sim e^{-2\operatorname{Im} I} = e^{\pi(R_f^2 - R_i^2)} = e^{\frac{1}{4}(A_f - A_i)} = e^{\Delta S_{B-H}}. \quad (33)$$

This result has the same functional form to that of neutral particles. It is also evident that the radiation spectrum is not precisely thermal. These results is consistent with that of the recent work [25–27].

5 Discussion and Conclusions

When $D = 0$, then Garfinkle-Horowitz-Stromingen dilaton black holes transit to Schwarzschild black hole. Under this condition, $R_i = r_i = 2M$, $R_f = r_f = 2(M-\omega)$, the tunneling rate of particles on event horizon of the Schwarzschild black hole can be obtained from (14)

$$\Gamma \sim e^{-2\operatorname{Im} I} = e^{\pi(r_f^2 - r_i^2)} = e^{\Delta S_{B-H}} = e^{-8\pi\omega(M-\frac{\omega}{2})}. \quad (34)$$

This equation is same to the known result [2].

In a word, we have investigated the tunneling processes of the massless particles and massive particles from Garfinkle-Horowitz-Stromingen dilaton black holes. The tunneling rates of these two different kinds of outgoing particles have the same functional form and all satisfy an underlying unitary theory. And the radiation spectrum is not précised thermal. So it is possible that the information is conserved during the radiation process.

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